# Performance Evaluation of the Second Order Digital Data-Aided Loop[1]

Tien M. Nguyen, *Senior Member, IEEE*
Jet Propulsion Laboratory
California Institute of Technology
4800 Oak Grove Dr, Pasadena, CA 91109

ABSTRACT Performance of the second order digital l>atr+-Aided Loop (DAL) is evaluated using the current available analog results. To utilize the analog results, both Impulse Invariant Transformation (I 1'1') and Linear Interpolat ion Transformation (LIT) techniques arc used in the approximation of the digital loop filter. The analytical results obtained from these transformation methods will be compared with the computer simulation results. The results obtained by LIT method arc in good agreement with computer simulation results. In addition, the paper will also investigate the impact of the DAL tracking phase jitter on the Bit Error Rate (BER) performance, rrnd the results arc then compared with the commonly used Costas loops, namely, Costas loop with matched filter and clock feed back in the arm filter and Costas loop with second order Butterworth filter in the arm filter. The analytical results demonstrate the superior performance of the digital DAL.

## 1.0 Introduction

In the past, the analog Jlatri-Aided Loop (DAL) has been proposed for space applications [1, 2]. The DAL can be employed by both suppressed carrier or residual carrier communication systems. For space applications, the residual carrier systems usually usc the subcarrier to separate the data from the residual carrier [3]. Basically, the DAL uses the power in the composite received signal sidebands to enhance the Signal-to-Noise Ratio (SNR) in the bandwidth of the carrier (or subcarr icr) tracking. The composite received signal used in the DAL can consist of the carrier and data, or carrier and data modulated subcarrier for suppressed or residual carrier system, respectively [2]. Furthermore, for residual carrier systems, the DAL can also be used for subcarrier tracking where the composite received signal is subcarrier and data [3, Chapters 5-6]. Because of the advance in digital signal processing technology, the DAL can be easily implemented in a single Digital Signal Processing (DSP) chip. 'J'his has motivated the usc of the digital DAL for space applications where subcarrier tracking is required to be done with great accuracy. Figure 1 shows a simplified block diagram of the digital DAL. The digital loop filter, F(z), shown in this figure is of the second order type, hence the name second order digital DAL.

The performance of the first and second order analog DAL has been analyzed thoroughly by Simon and Springett [2]. However, the results for the second order loop arc only applicable to the second order analog loop filter. This paper attempts to usc the current available results provided in [2] to derive the tracking rms phase error (or tracking phase jitter) for the second order digital DAL, and assesses the impact of this phase jitter on the Bit Err or Rate (BER) performance. The results of the phase jitter obtained by the computer simulation for the digital DAL will also be presented and compared with the theoretical results. Furthermore, the BER performance for systems using DAL will be compared against those employing Costas loops.

## 2.0 Derivation of Tracking Phase Jilter

### 2.1. Current Results for the Analog DAL

A simplified block diagram for the analog DAL is shown in Figure 2. This loop has been analyzed in [1-2]. For the second order DAL, the loop filter F(s) is given by

$$F(s) = \frac{1 + s\tau_2}{1 + s\tau_1} \tag{1}$$

where $\tau_1$ and $\tau_2$ arc the time constants of the second order loop filter. When the Loop Signal-to-Noise Ratio (LSNR) is large and the bit SNR is greater than 4 dB with the tracking phase jitter less than 15° (or $\pi/12$), the variance of the tracking phase jitter can be shown to have the following form [2]

$$\sigma_\phi^2 = \left[1 + \frac{1 - F_1}{r}\right]\left[\left(\frac{1 + r}{r}\right)\rho\right]^{-1} \tag{2}$$

where $F_1$ is defined as the time constant $\tau_2$-to-$\tau_1$ ratio and the parameter r is given by

[1] The work described in this paper was carried out at the Jet I'repulsion laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

$$r = \sqrt{P_D}\, K \tau_2 F_1 \operatorname{erf}(\sqrt{R_s}), \qquad (3)$$

the parameters $P_D$ and $R_s$ denote the total received signal power and Bit SNR (BSNR), respectively, and K in Eq. (3) denotes the loop gain. Finally, the parameter $\rho$ in Eq. (2) is given by

$$\rho = R_s \delta \operatorname{erf}^2(\sqrt{R_s}), \qquad (4)$$

Note that Eq. (2) represents the linear approximation of the tracking phase jitter for the second order loop at high bit SNR and small phase jitter error. For large r, the phase jitter described by Eq. (2) will approach (1/p), which is the phase jitter for the linear first order loop, Thus, for large r, the parameter p becomes the loop SNR.

In this paper, one is interested in a digital DAL (see Figure 1) with a second order digital loop filter F(z) of the following type:

$$F(z) = A_1 + \frac{A_2}{z-1} \qquad (5)$$

where $A_1$ and $A_2$ are the coefficients of the digital filter. For typical deep space missions, e.g. Mars Observer and Cassini, the coefficients are given by Al = 0.25 and $A_2$ = 0.031?,5, In order to use the analog results presented above, one must find the equivalent loop filter in the S-domain.

### 2.2. Equivalent Loop Filter in the S-Domain

From Eq. (5), the loop filter F(z) in the discrete domain can be rewritten as

$$F(z) = K_f \left[\frac{1-\gamma_2 z}{1-\gamma_1 z}\right] \qquad (6)$$

where in this case

$$\gamma_1 = 1,\ \gamma_2 = \frac{A_1}{K_f},\ K_f = A_1 - A_2 \qquad (7)$$

The goal is to find whether there exists appropriate values for $\tau_1$ and $\tau_2$ in Eq. (1) that can be used to approximate the digital loop filter in Eq. (6) so that the analog results presented in Section 2.1 can be applied, Using the LIT techniques [4], one can replace s in Eq. (1) by

$$\left(\frac{2}{T}\right)\left[\frac{z-1}{z+1}\right] \qquad (8)$$

to get an approximated digital loop filler for F(z), $F_{Ap}(z)$. The parameter T in Eq. (8) denotes the nominal sampling period, For typical deep space missions, the nominal sampling period is $(8 \times 10^3)^{-1}$ sec. Let $a_1$ and $a_2$ be defined as

$$a_1 = \frac{2\tau_1}{T},\quad a_2 = \frac{2\tau_2}{T} \qquad (9)$$

then the approximated digital loop filter $F_{Ap}(z)$ of F(z) using LIT transformation technique can be written in terms of al and $a_2$ as

$$F_{Ap}(z) = \left(\frac{a_2-1}{a_1-1}\right)\left[1-\left(\frac{a_2+1}{a_2-1}\right)z\right]\left[1-\left(\frac{a_1+1}{a_1-1}\right)z\right]^{-1} \qquad (10)$$

Next, one would want to map Eq. (10) into Eq. (6). By equaling the coefficients between Eqs. (10) and (6) one obtains the appropriate time constants $\tau_1$ and $\tau_2$ for the corresponding analog filter that represents F(z) in the s-domain, namely,

$$\tau_1 = T\left[\frac{1}{K_f(\gamma_2-1)} + \frac{1}{2}\right],\quad \tau_2 = \frac{T}{2}\left[\frac{\gamma_2+1}{\gamma_2-1}\right] \qquad (11)$$

Since one wants the approximated second order digital loop filter $F_{AP}(z)$ to be the same as F(z) described in Eq. (6), the values $\tau_1$ and $\tau_2$ calculated from Eq (11) must be selected in such a way that the following condition is satisfied

$$\gamma_1 = \left(\frac{a_1+1}{a_1-1}\right) \approx 1 \qquad (12)$$

As an example, for typical deep space missions, one obtains:

$$\tau_1 = 4.0585 \times 10^{-3}\,\mathrm{sec},\ \tau_2 = 9.366 \times 10^{-4}\,\mathrm{sec} \qquad (13)$$

Using the calculated values for $\tau_1$ and $\tau_2$ in Eq. (1 3), one wishes to verify the condition in Eq. (12). By substituting these values into Eq. (12) one gets $\gamma_1$ = 1.03, which is approximately equal to 1, Therefore, for deep space applications, using the time constants found in Eq. (1 3), one can approximate the discrete loop filter F(z) in Eq. (6) by Eq. (1) in the s-domain. Thus, using the LIT method, the approximated analog loop filter for the digital filter described by Eq. (6) is found to be:

$$F(s) = \frac{1+s\tau_2}{1+s\tau_1} = \frac{1+(9.3662587\times 10^{-4})s}{1+(4.05850\times 10^{-3})s} \qquad (14)$$

To preserve the transient response of the analog loop filter F(s) in the discrete domain, the actual representation of Eq. (1) in the z-domain can be derived by using IIT method. This is derived as follows. First, the impulse response f(t) of the analog loop filter is found by taking the inverse Laplace transform of F(s). The desired impulse response f(n) for the digital loop filter then can be found by sampling f(t) at each sampling interval T, i.e., f(n) = f(t = nT). The actual loop filter, $F_{Ac}(z)$, in the discrete domain is found by taking the Z-transform of f(n), namely [5],

$$F_{AC}(z) = \frac{\alpha_0 - \alpha_1 e^{-\alpha_2} z^{-1}}{1 - z^{-1} e^{-\alpha_2}} \tag{15}$$

where the parameters $a_0$, a, and $a_2$ arc given by

$$\alpha_0 = \frac{\tau_2 + 1}{\tau_1} - \frac{\tau_2}{\tau_1^2},\ \alpha_1 = \frac{1}{F_1},\ \alpha_2 = \frac{T}{\tau_1} \tag{16}$$

To approximate the digital loop filter expressed by Eq. (5) in the analog domain onc rewrites Eq. (5) as follows

$$F(z) = K_f \left[ \frac{\gamma_2 - z^{-1}}{1 - z^{-1}} \right] \tag{17}$$

where $K_f$ rrnd $\gamma_2$ arc given by Eq. (7) and $\gamma_1 = 1$. By comparing the coefficients of Eqs. (15) and (17), onc can determine the time constants $\tau_{1Ac}$ and $\tau_{2Ac}$ for the corresponding, analog loop filter, $F_{Ac}(s)$, that represents an approximation using IIT transformation for the digital loop filter described by Eq. (6). Comparing the coefficients between Eqs. (15) and (17), onc gets

$$\left( \frac{\tau_{2Ac} + 1}{\tau_{1Ac}} - \frac{\tau_{2Ac}}{\tau_{1Ac}^2} \right) = \gamma_2, \text{ and } e^{-\frac{T}{\tau_{1Ac}}} = 1 \tag{18}$$

"Ike solutions to Eq. (18) should satisfy the constraint

$$\frac{\tau_{2Ac}}{\tau_{1Ac}} e^{\frac{-1}{\tau_{1Ac}}} = 1 \tag{19}$$

As an example, the calculated time constants using IIT for typical deep space applications arc, from Eqs. (18) and (19),

$$\tau_{1Ac} = 0.1389 \text{ See, } \tau_{2Ac} = 0.1357 \text{ sec.} \tag{20}$$

Plots of the variance tracking jitter using IIT and LIT arc shown in Figure 3. The computer simulation results arc also shown for comparison and verification purpose. LIT technique was found to bc the best and will bc used later for BER calculation.

## 3. BER Performance

In this section onc will assume that the bit tracking is perfect in the bit synchronizer. When the two-sided loop bandwidth of the digital DAL (or the analog Costas loop) is small relative to the in coming data rate, then the phase error $\phi$ can bc considered to he constant for many bit periods. Under these conditions, the conditional error probability is given by [2]

$$P(e/\phi) = \frac{1}{2} \operatorname{erfc}(\sqrt{R_s} \cos(\phi)) \tag{21}$$

Again, $R_S$ denotes the bit SNR. The average error probability is then obtained by averaging Eq. (21) over the probability density function (pdf) P(f) of the phase error [2]

$$P_{Ac} = \int_{-\pi/2}^{\pi/2} P(e/\phi) P(\phi) d\phi \tag{22}$$

For digital DAL operates at high bit SNR ($R_s >$ 4 dB) and small phase error ($\phi < 15^0$), using Reference [2], the pdf for the phase error can be shown to have the following form

$$P(\phi) = e^{\alpha \cos(\phi)} \left[ \int_{-\pi/2}^{\pi/2} e^{\alpha \cos(\phi)} d\phi \right]^{-1} \tag{23}$$

where

$$\alpha = \sigma_\phi^{-2} \tag{24}$$

and $\sigma_\phi^2$ is given by Eq. (2). The average BER performance for the digital DAL loop and the results are calculated and plotted in Figure 4. For Costa loop with matched filter and clock feed back in the arm filter, and second order Butterworth in the arm filter, the results can bc found in [3, Chapter 3]. The results arc also plotted in Figure 4 for comparison purpose.

## 4.0 Conclusion

The analytical model employing the LIT method can bc used to predict the tracking jitter of the second order digital DAL for all data rates. Using the derived tracking phase jitter, onc can determine the BER performance of the second order digital suppressed-carrier DAL. Numerical results show that the digital DAL outperforms the commonly used Costas loops

## Acknowledgement

"Thanks arc due to D. Hansen for providing the computer simulation results, Drs. V. Vilnrotter and S. Hinedi for their invaluable comments and

suggestions during the review of this work, and A. Kermode and B. Charny for their support.

## References


[1] Lindsey, W. C., M K Simon, "Data Aided Carrier Tracking loops," IEEE Trans on ComTech, Vol. COM-19, No ?. April 1971

[2] M. K. Simon, J C. Springett, " The Theory, Design, and Operation of the Suppressed Carrier Data-Aided Tracking Receiver," Tech Rep. 32-1583, June IS, 1973, JPL, CA.

[3] J. Yuen, "Deep space Telecommunication Systems Engineering," Plenum, N Y., 1983

[4 E. 1. Jury, Theory and Application of the Z-Transform Method, John Wiley & Son, 1986.

[5 J W. Heller, "A Digital Filter Implementation of the Deep space Transponder PLL Integrator," TDA Progress Report 42-66, 1981 > Jet Propulsion Laboratory, California


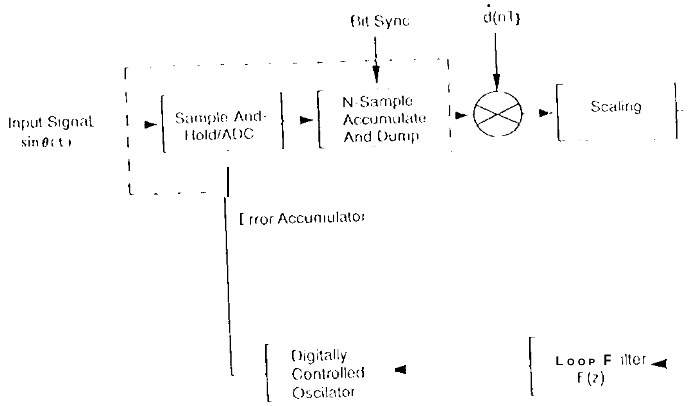


Figure 1 Simplified Block Diagram for the Digital Data-Aided Loop

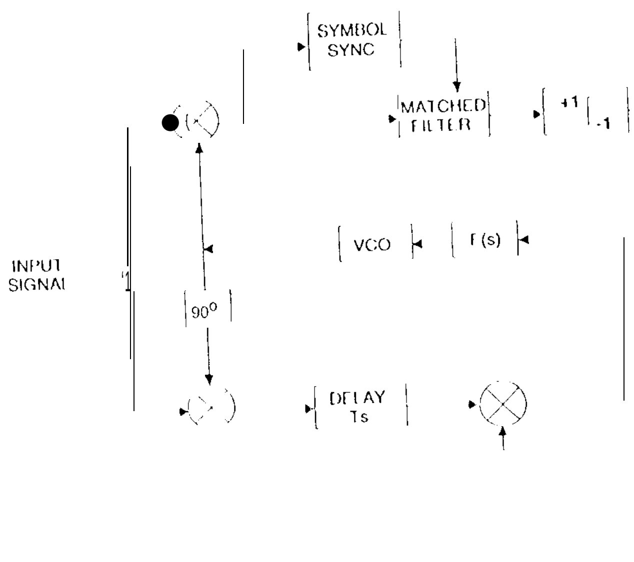


FIGURE 2 Simplified Block Diagram for the Analog Data-Aided Loop

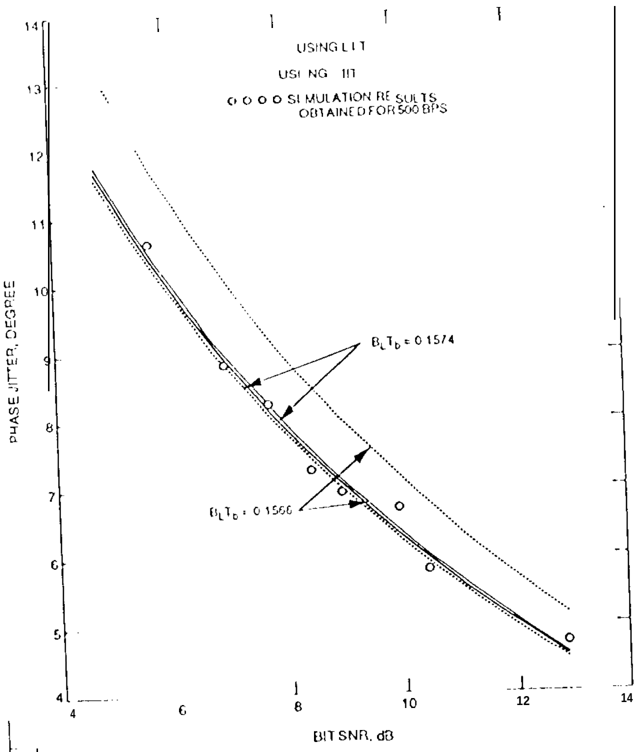


Figure 3 A Comparison Between IIT and LIT

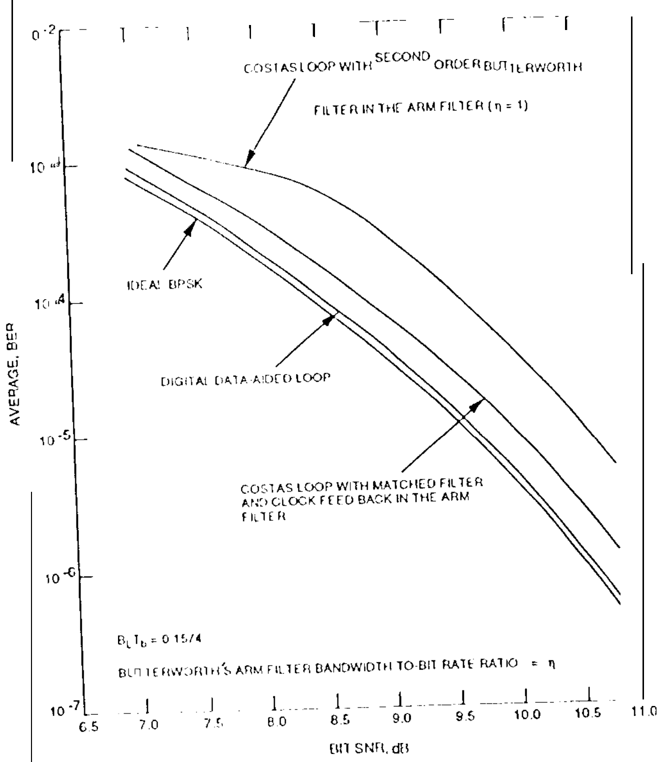


Figure 4. Average Probability of Error for Digital DAL and Costas Loop